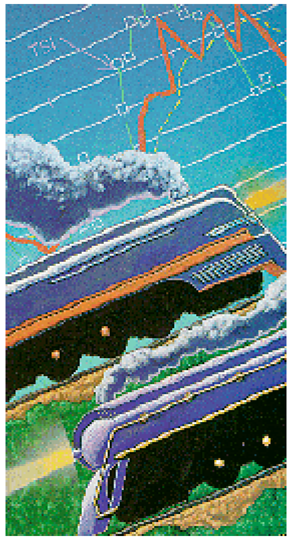
Stochastic RSI And Dynamic Momentum Index

by Tushar Chande and Stanley Kroll



Here, a relative strength index (RSI) tutorial clarifies the basics of the time-honored indicator. The tutorial also introduces new variants that STOCKS & COMMODITIES contributor Tushar Chande and Kroll, Chande & Co.'s Stanley Kroll call net momentum oscillator and the stochastic RSI, which are better at flagging overbought and oversold conditions than RSI itself. Also introduced and examined is a variable-length RSI called the dynamic momentum index (DMI), which we indexed to market volatility. The new variants, Chande explains, extend the scope and power of momentum indices, as we will see.

Many articles in STOCKS & COMMODITIES have described J. Welles Wilder's relative strength index (RSI) and its derivatives. One recent example was the relative momentum index (RMI), which was discussed by Roger Altman. In another, William Blau wrote about the true strength index (TSI), a double-smoothed version of RSI that is identical to the RSI except for a scaling constant.

Each RSI derivative is a valuable extension of the approach. However, details of RSI development were not shown in either derivation, and thus, it is sometimes difficult to determine what RSI and its derivatives are measuring. They are, of course, measuring momentum. But that is only part of the story.

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We will take a tutorial look at RSI and its derivatives, then break down RSI and reassemble it so we know how it works. After that we will do the same with the derivatives. First, we will discuss a net momentum oscillator and a stochastic RSI, which are better at flagging extremes in prices than RSI. Then we will index the RSI to price volatility. This leads to a variable-length dynamic momentum index (DMI) that automatically adjusts to market action. The new RSI variants extend the scope and power of momentum indices.

THE BASICS OF MOMENTUM

In technical analysis, we define momentum as the difference between two prices. The time period between the two prices varies. The two prices can be as little as one period apart, while the period can be any time length from minutes to months. Most commonly, we use one trading day as the unit of time. With a one day time period, the closing prices (C) define momentum as shown here.

(1) $\text{Momentum} = C_{today} - C_{yesterday}$

Because today's close could be above or below yesterday's close, momentum can be positive or negative. We will call this relative momentum. We can define absolute momentum by ignoring the sign of daily momentum as:

(2) $|\text{Momentum}| = |C_{today} - C_{yesterday}|$

The values of absolute momentum are always positive, while the values of relative momentum can be positive or negative. Now let us perform a short exercise using five trading days. We will use values for the Standard & Poor's 500 index from November 29, 1991, to December 15, 1991, from the sidebar, "The RSI and the RMI." Now let us calculate the daily relative momentum and absolute momentum. The calculations can be seen in Figure 1.

Note that the sum of the relative momentum over four days is equal to the momentum over four days. This occurs because the intermediate terms cancel each other when we add them (see Equation 3). In Equation 3, the subscripts refer to the number of days back from today (zero).

(3) $(C_0 - C_4) = (C_o - C_1) + (C_1 - C_2) + (C_2 - C_3) + (C_3 - C_4)$

Now, continue this exercise in a modified format. This time, we will separate the daily momentum into values for days when the market closes up or closes down. In each case, the momentum values will be positive. On an up day, the momentum is today's close minus yesterday's close. We calculate momentum as the close of yesterday minus the close of today on down days (see the results in Figure 2).

Figure 2 gives us two key results. Let S_u and S_d be the sum of the up day and down day momentum over a given number of days. The relative momentum over four days is the difference between the up day momentum and down day momentum over four days (2.17 = 6.18 - 4.01). In addition, the absolute momentum is the sum of the up day momentum and down day momentum (10.19 = 6.18 + 4.01). We can summarize these important results in the equations below.

(4) $\text{Momentum} = S_u - S_d$

(5) $|\text{Momentum}| = S_u + S_d$

We can solve Equations 4 and 5 for S_u using algebra. Add the two equations and gather terms to obtain an equation for S_u as shown below in Equation 6.

(6) $S_u = 0.5*(\text{Momentum} + |\text{Momentum}|)$

RELATIVE MOMENTUM AND ABSOLUTE MOMENTUM

Date	S&P500	Relative Momentum	Absolute Momentum
1/29/91	375.22		
12/02/91	381.40	6.18	6.18
12/03/91	380.96	–0.44	0.44
12/04/91	380.07	–0.89	0.89
12/05/91	377.39	–2.68	2.68
Total	2.17	2.17	10.19

Notes:

1. Relative momentum = close of today - close of yesterday

2. Absolute momentum = close of today - close of yesterday

3. 4-day momentum = 377.39-375.22 = 2.17

FIGURE 1: *Relative momentum can be positive or negative, while absolute momentum is always positive.*

MOMENTUM SEPARATED FOR UP AND DOWN DAYS

Date	S&P 500	Rlative Momentum	Absolute Momentum	Up Day Momentum	Down Day Momentum
11/29/91	375.22				
12/02/92	381.40	6.18	6.18	6.18	0.00
12/03/91	380.96	–0.44	0.44	0.00	0.44
2/04/91	380.07	–0.89	0.89	0.00	0.89
12/05/91	377.39	–2.68	2.68	0.00	2.68
Total	377.39=	2.17	10.19	6.18	4.01
	375.22=	2.17		(S_u)	(S_d)

FIGURE 2: *Define S_u and S_d as a sum of the up day momentum and down day momentum.*

CALCULATING FOUR-DAY RSI USING SUM OF UP DAY AND DOWN DAY MOMENTUM

Date	S&P 500	Up Day Momentum (S_u)	Down Day Momentum (S_d)	RSI
11/29/91	375.22			
12/02/92	381.40	6.18	0.00	
12/03/91	380.96	0.00	0.44	
2/04/91	380.07	0.00	0.89	
12/05/91	377.39	0.00	2.68	
Total		6.18	4.01	60.65

Note:

RSI = 100*(6.18)/(6.18+4.01) = 60.64769382

FIGURE 3: *Here, the four-day RSI is calculated using up and down momentum.*

This result can be verified from the data in Figure 2 (S_u = 0.5*(10.19+2.17) = 6.18). We will use these results to define Wilder's RSI.

RELATIVE STRENGTH INDEX (RSI)

The relative strength index measures the proportion of absolute momentum change over a given period due to momentum on up days. This will become clear from the definition shown in Equation 7:

(7) RSI = 100*(RS/(1+RS))

Here, relative strength (RS) is the ratio of average momentum over the last *n* days on up days versus down days (A_u and A_d, respectively). The revised RS is shown in Equation 8.

(8) RS = A_u/A_d

Because the number of days in both averages is the same, we can multiply both averages by the number of days in the average. This converts the averages into the sum of the momentum on up days and down days. Thus, we can rewrite Equation 8 in terms of S_u and S_d as shown below.

(9) RS = S_u/S_d

We can now rewrite the RSI definition using Equation 9 and substituting for RS. This gives us an RSI definition in terms of the sum of the momentum on up days and down days.

(10) RSI = 100*(S_u)/(S_u+ S_d)

We can continue our process of adjustment to reflect the relationship between relative momentum and absolute momentum, S_u and S_d. We can rewrite RSI using absolute momentum and relative momentum as defined in Equations 4, 5 and 6.

$$\text{RSI} = \frac{100\left(0.5\left(\text{Momentum} + |\text{Momentum}|\right)\right)}{|\text{Momentum}|}$$

$$\text{RSI} = \frac{50\left(\text{Momentum} + |\text{Momentum}|\right)}{|\text{Momentum}|} \qquad (11)$$

We have found something important in Equation 11, tying RSI into relative and absolute momentum. This relationship is not obvious in the original definition of RSI. Thus, over *x* days, the RSI is the proportion of absolute momentum due to up days, and so, the x-day RSI is equivalent to the x-day momentum. We can now physically see what RSI is trying to depict, and so, we will illustrate these ideas by calculating a four-day RSI using the data in Figure 3.

We will redo the four-day RSI calculations using the relative and absolute momentum (Figure 4). I reiterate that the four-day relative momentum is simply the four-day momentum, and so, the four-day RSI is closely related to the four-day momentum. Similarly, the 14-day RSI will be closely related to 14-day momentum. I will complete our discussion by showing that we get the same value using either method. Now it is easier to understand what the RSI is trying to capture by using the ideas of relative and absolute momentum. We will discuss the smoothing of RSI values later. We should also remember that RSI uses

CALCULATING FOUR-DAY RSI USING RELATIVE AND ABSOLUTE MOMENTUM

Date	S&P 500	Relative Momentum	Absolute Momentum	RSI
11/29/91	375.22			
12/02/92	381.40	6.18	6.18	
12/03/91	380.96	–0.44	0.44	
2/04/91	380.07	–0.89	0.89	
12/05/91	377.39	–2.68	2.68	
Total		2.17	10.19	60.65

Note:

RSI = 50*(2.17+10.19)/(10.19) = 60.64769382

FIGURE 4: *Here, the four-day RSI is calculated using relative and absolute momentum.*

CALCULATING THE 20-DAY RMI WITH FIVE-DAY LOOKBACK USING RELATIVE AND ABSOLUTE MOMENTUM

Date	S&P500	Momentum	|Momentum|
29-Nov-91	375.22		
02-Dec-91	381.40		
03-Dec-91	380.96		
04-Dec-91	380.07		
05-Dec-91	377.39		
06-Dec-91	379.10		3.88
09-Dec-91	378.26		3.14
10-Dec-91	377.90		3.06
11-Dec-91	377.70		2.37
12-Dec-91	381.55		4.16
13-Dec-91	384.47		5.37
16-Dec-91	384.46		6.20
17-Dec-91	382.74		4.84
18-Dec-91	383.48		5.78
19-Dec-91	382.52		0.97
20-Dec-91	387.04		2.57
23-Dec-91	396.82		12.36
24-Dec-91	399.33		16.59
25-Dec-91	404.84		21.36
26-Dec-91	406.46		23.94
27-Dec-91	415.14	39.92	28.10
30-Dec-91	417.09	35.69	20.27
31-Dec-91	417.26	36.30	17.93
01-Jan-92	419.34	39.27	14.50
02-Jan-92	417.96	40.57	11.50
Total		**191.75**	**208.89**
	Su	200.32	
	Sd	8.57	
	RMI	95.90	

Notes:

1. 20-day momentum = (415.14-375.22) = 39.92
2. S_u = 0.5*(Momentum + |Momentum|)
3. S_d = |Momentum| - S_u
2. RMI = 50*(191.75+208.89))/208.89 = 95.89736225

FIGURE 5: The relative momentum index is calculated based on a 20-day period using a five-day momentum.

the daily difference in calculating momentum; its lookback period for the momentum calculations is one day.

RELATIVE MOMENTUM INDEX (RMI)

The relative momentum index extends the RSI approach by increasing the number of days in the lookback period of calculating momentum. Thus, instead of taking the difference between today's close and yesterday's close, we can use the close x days ago. Roger Altman used a five-day lookback to calculate the momentum. All the other steps in calculating RMI are the same as in calculating RSI.

If we break down the RMI calculations we will get all the equations we derived for RSI. In the end, we can write RMI in terms of relative and absolute momentum the same way we did for RSI. The only difference is that we are using the sum of momentum over five days instead of one day. In "Relative momentum index: modifying RSI," Altman defined RMI in the same way as RSI:

(12) RMI= 100*(RM/(1+RM))

Here, the term RM is the ratio of the sum of the up day momentum to the down day momentum over *x* days. We saw above that the four-day relative momentum was simply the four-day momentum. If we used a 20- day momentum summed over five days to calculate relative momentum, then we would get the following:

(13) Momentum = $(C_{25} - C_5)+(C_{24} - C_4)+(C_{23} - C_3)+(C_{22} - C_2)+ (C_{21} - C_1)$

The absolute momentum is calculated using a five-day lookback. All the relationships between relative, absolute momentum and sum of the up day and down day momentum apply here. Hence, we can rewrite RM and RMI using S_u and S_d.

(14) RM = S_u/S_d

(15) RMI = $100*(S_u)/(S_u + S_d)$

I will illustrate the RMI calculation using absolute and relative momentum. You can see the calculations using up day momentum and down day momentum in the sidebar. We will use Equation 15 given above.

Figure 5 supports our arguments that we can write RMI in terms of absolute and relative momentum taken over x-day lookback.

I recommend calculating an unsmoothed RSI or RMI using all data in the *x*-day lookback period. You can then smooth the calculations with a simple or exponential moving average.

The totals for 20-day momentum up and down are 200.32 and 8.57. We can verify that relative momentum is the difference between the 20-day momentum of up days and down days, because 200.32 - 8.57=191.75; further, 200.32+8.57=208.89, the absolute momentum. We can summarize these relationships for RMI in the following equations.

$$\mathrm{RSI} = \frac{100\left(0.5\left(\mathrm{Momentum} + \left|\mathrm{Momentum}\right|\right)\right)}{\left|\mathrm{Momentum}\right|}$$

$$\mathrm{RSI} = \frac{50\left(\mathrm{Momentum} + \left|\mathrm{Momentum}\right|\right)}{\left|\mathrm{Momentum}\right|} \quad (16)$$

Equation 16 represents RMI with *x*-day lookback and y-day calculations. Like the RSI, the RMI is related to momentum summed over equivalent periods. It also expresses the proportion of absolute momentum due to up days.

SMOOTHING RSI AND RMI

The smoothing mechanism in RSI and RMI is somewhat irregular. For an *x*-day RSI or RMI, l/*x* of the sum of up momentum and down momentum is deducted from yesterday's value. Then today's value of the up momentum and down momentum is added to this modified value of yesterday's total. This approach does not respond quickly when we start calculating over a period of rapid market movement.

I like to use all 20-day data to calculate successive values of RSI or RMI. The irregular smoothing approach was convenient for calculations by hand, but we need not use it with computers commonly used today. I recommend calculating an unsmoothed RSI or RMI using all data in the *x*-day lookback period. You can then smooth the calculations with a simple or exponential moving average.

I will show the effect of this smoothing using data for the February 1993 heating oil #2 futures contract. Figure 6 shows the daily close to illustrate the general trend, while Figure 7 shows the RMI smoothed using Wilder'smethod together with the unsmoothed RMI The smoothed RMI DIverged significantly from the unsmoothed RMI and masked the rapid deterioration in the market. In Figure 8 I smoothed the RMI with a five-period simple moving average. The unsmoothed values plus a simple moving average provides a better picture of market action.

A quick glance at Equation 8 shows that the ratio RS uses a simple moving average of up day and down day momentum. William Blau suggested using two exponential moving averages to smooth the up day and down day momentum. He called it the double-smoothed RSI or $\mathrm{DRSI}_{y,z}$ where y and z are the exponential moving averages.

The double-smoothed values are virtually the same as taking a second moving average of single-smoothed RSI, provided we are using exponential moving averages for both steps. The DRSI values are usually significantly different from single-smoothed RSI values. By design, they show a greater sensitivity to price changes. The length of your trading horizon will influence your preference for double- or single-smoothed RSI.

NET MOMENTUM OSCILLATOR

The Nmo is another extension of our RSI tables. As Roger Altman astutely pointed out, the RSI is not always easy to interpret because it does not oscillate evenly between overbought and oversold regions. The problem arises because the number of days in RSI calculations is constant. The RSI may not reach 30% or 70% when calculated over a fixed number of days. A better approach is to use the ratio of the momentum in up days versus momentum in down days. I like to use a net momentum oscillator (NMO) as shown below:

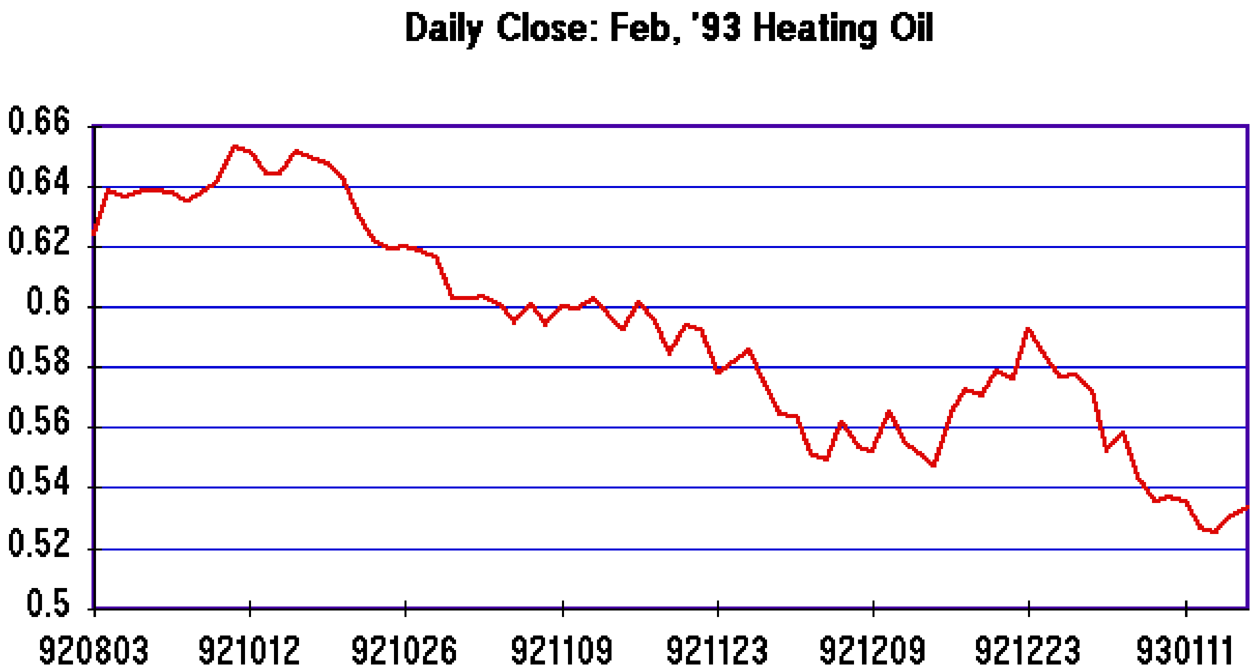

FIGURE 6: DAILY CLOSE FEBRUARY 1993 HEATING OIL. The closing price was in a clear downtrend during late 1992.

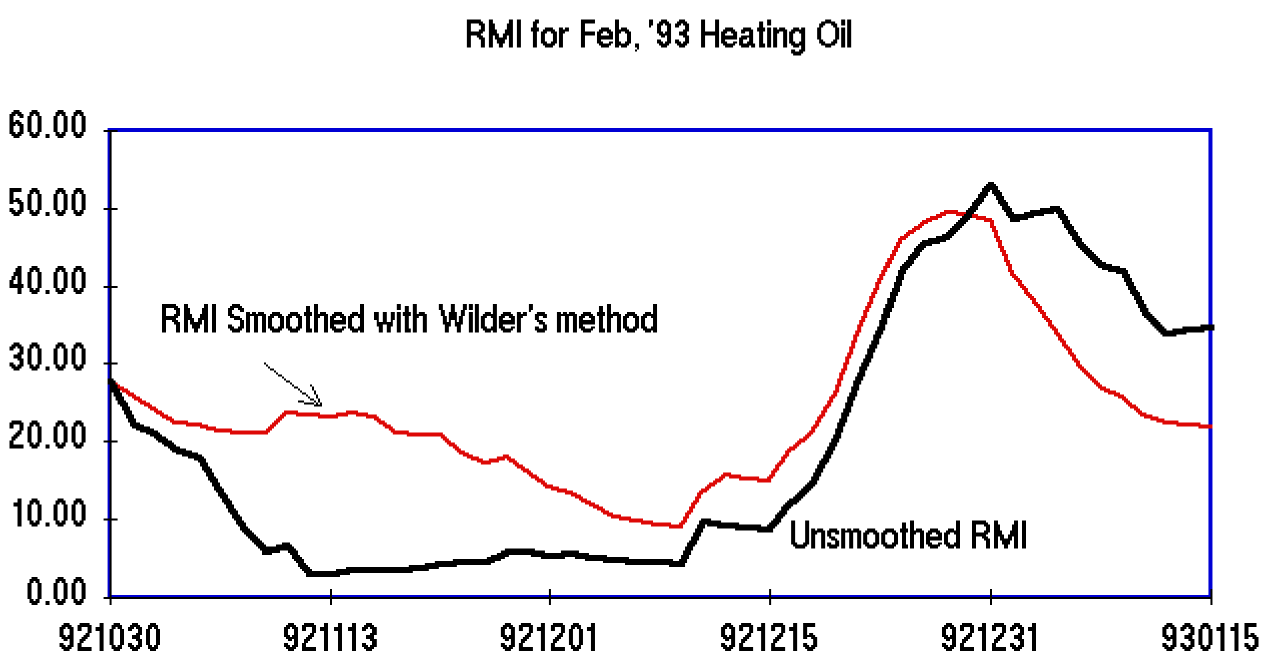

FIGURE 7: RELATIVE MOMENTUM INDEX (RMI). The RMI is presented unsmoothed and smoothed using Wilder's method. The unsmoothed version indicates market weakness by the low readings in early November.

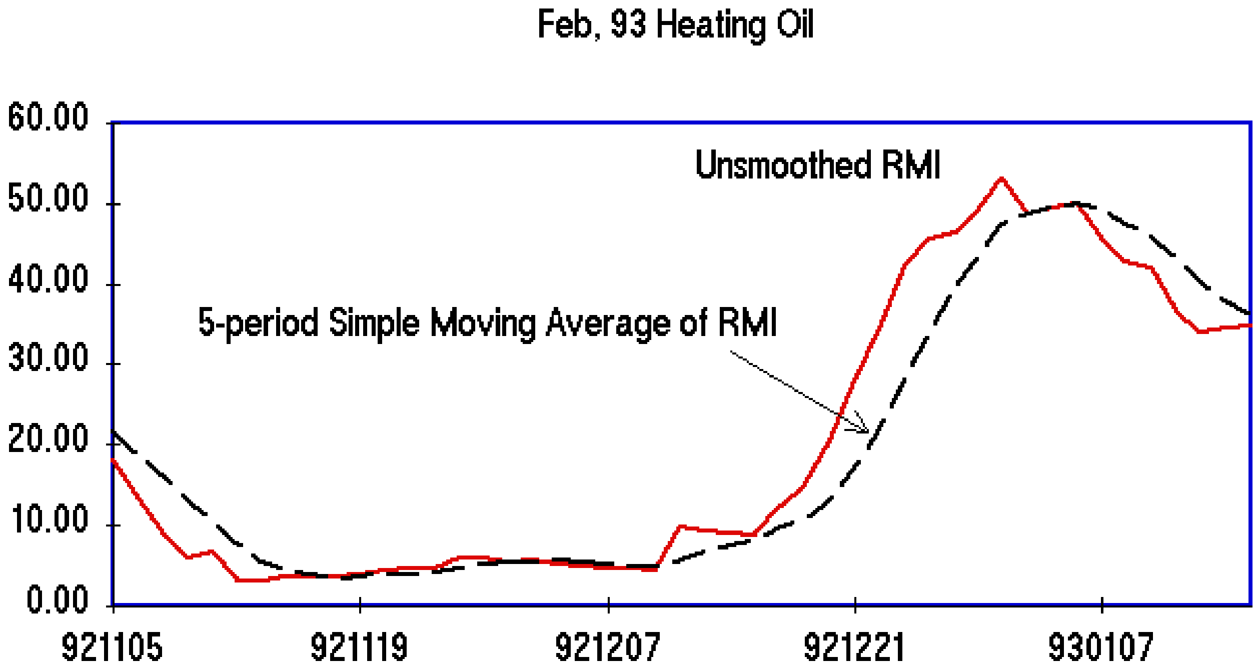

FIGURE 8: RELATIVE MOMENTUM INDEX (RMI). The unsmoothed RMI along with a five-period simple moving average can provide a better picture of market action.

(17) Net Momentum Oscillator = $100*(S_u - S_d)/(S_u + S_d)$

The net momentum oscillator takes the difference between the up day momentum and down day momentum. It then expresses this difference as a ratio of the sum of up and down day momentum.

The primary benefit is that we can define overbought and oversold levels at +50 and -50 where the ratio of S_u to S_d is 3:1 or 1:3. You can find by simple substitution that a ratio of 3:1 corresponds to a value of +50. This means that the up day momentum is three times the down day momentum. The reverse is also true. If the up day momentum is only a third the down day momentum, then this oscillator has a value of -50. Note that RSI does not give consistent overbought and oversold signals. The NMO can overcome this limitation by using the ratio of S_u to S_d.

In Figure 9, I tabulated the values of the S_u to S_d ratio and the corresponding unsmoothed values of RSI, RMI and NMO. This table shows that the S_u to S_d ratio is 2.33 for a RSI value of 70. A RSI value of 30 corresponds to the inverse of the S_U to S_d ratio for the RSI value of 70. Thus, S_u/S_d value of 0.43 gives a RSI value of 70. You can now pick a S_u/S_d value you want to define overbought and oversold levels.

Another benefit of NMO is that we get positive and negative values. Thus, it is easy to see whether the market action is mostly up or down. In Figure 10, I plotted the 20-day NMO for heating oil data. Note the similarity to the unsmoothed RMI, since the lookback period was also five days for NMO calculations. The difference is the *y*-axis scale, which shows negative values. The net momentum was negative, showing the market was in a downtrend, as seen in Figure 6.

A better way to visualize the net momentum oscillator is to rewrite Equation 16 in terms of relative momentum and absolute momentum. Using the definitions given earlier, we have the result that:

$$\text{Net Momentum Oscillator} = 100\left(\frac{\text{Momentum}}{|\text{Momentum}|}\right) \quad (18)$$

Equation 18 reveals that the NMO simply shows the relative momentum as a fraction of the absolute momentum changes over the desired period. The purpose of our exercise in defining relative and absolute momentum is now clear.

The NMO is similar but not identical to the true strength index (TSI) described by William Blau. Blau's TSI uses double-smoothed exponential averages for the relative and absolute momentum. Thus, its values differ significantly from the NMO and will show greater sensitivity to price changes. On the other hand, because of the exponential smoothing, the TSI rarely approaches extremes of -100 and +100, and so, the NMO can show the extremes more readily. The choice of NMO or TSI will depend on your trading horizon and your preference for indicator sensitivity. Figure 11 shows a comparison of NMO and TSI for February 1993 heating oil. The TSI has 20-day exponential averages double smoothed by three-day exponential averages. We smoothed the 20-day unsmoothed NMO with a five-day exponential moving average. The daily closing prices can be seen in Figure 6. The TSI had greater sensitivity to prices changes and the NMO easily showed the oversold condition.

I prefer to use all previous data to calculate new values of NMO. We can then smooth NMO using a simple or exponential moving average to give a better representation of market action.

Those of you with technical analysis software can use the following equation to calculate NMO. This

VALUES OF S_U/S_D AND CORRESPONDING RSI, RMI AND NMO

S_u/S_d	RSI	RMI	NMO
.00	.00	.00	–100.00
.05	5.00	5.00	–90.00
.11	10.00	10.00	–80.00
.18	15.00	15.00	–70.00
.25	20.00	20.00	–60.00
.33	25.00	25.00	–50.00
.43	30.00	30.00	–40.00
.54	35.00	35.00	–30.00
.67	40.00	40.00	–20.00
.82	45.00	45.00	–10.00
1.00	50.00	50.00	.00
1.22	55.00	55.00	10.00
1.50	60.00	60.00	20.00
1.86	65.00	65.00	30.00
2.33	70.00	70.00	40.00
3.00	75.00	75.00	50.00
4.00	80.00	80.00	60.00
5.67	85.00	85.00	70.00
9.00	90.00	90.00	80.00
19.00	95.00	95.00	90.00
Infinite	100.00	100.00	100.00

FIGURE 9: *The table demonstrates the relative values for both overbought and oversold conditions for each oscillator.*

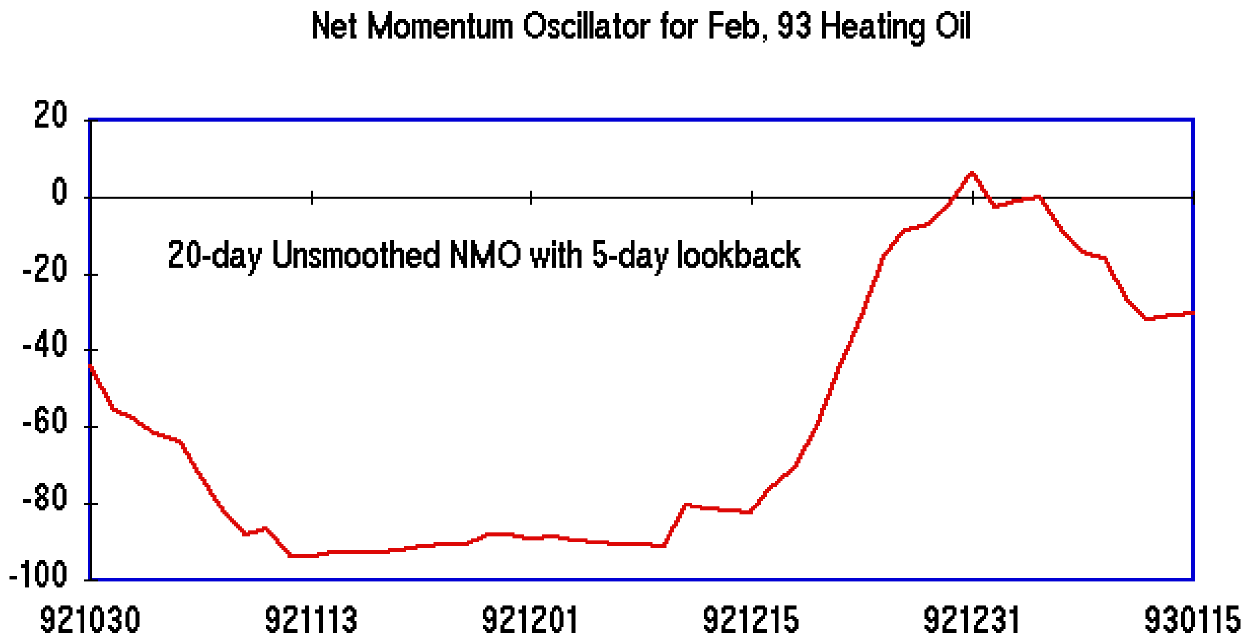

FIGURE 10: NET MOMENTUM OSCILLATOR FOR FEB 1993 HEATING OIL. The 20-day net momentum oscillator is similar to the relative momentum index.

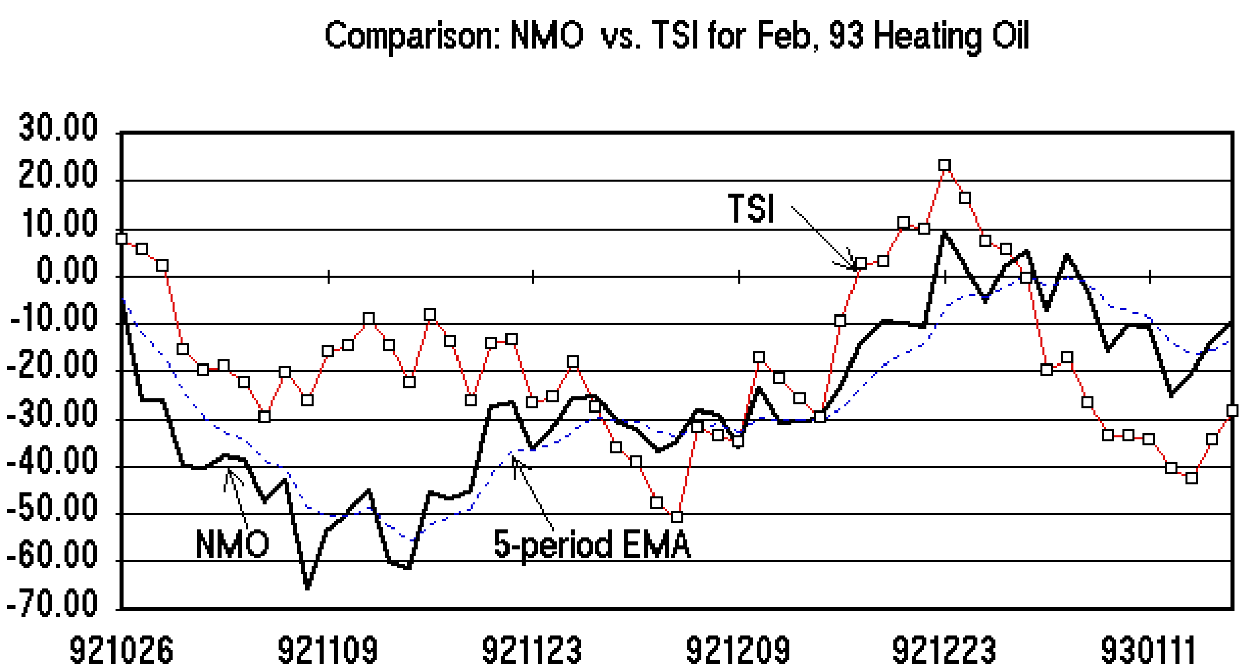

FIGURE 11: NET MOMENTUM OSCILLATOR VERSUS TRUE STRENGTH INDEX. The true strength index shows a great sensitivity to price changes.

relationship follows from the definitions of RSI and NMO. Your actual values will depend on the smoothing mechanism for RSI that your package uses.

(19) NMO = (2*RSI - 100)

STOCHASTIC RSI OSCILLATOR

The problem of consistently finding overbought and oversold regions can also be solved another way. We can adopt the computational approach used in the stochastic oscillator to form a stochastic RSI. This stochastic RSI is even better than NMO at identifying overbought/oversold regions. It gives consistent indicator values under overbought and oversold conditions.

We define the unsmoothed stochastic RSI as follows:

$$\text{StochRSI} = \frac{(RSI_{Today} - RSI_{Low})}{(RSI_{High} - RSI_{Low})} \quad (20)$$

Here, RSI_{High} and RSI_{Low} are the highest and lowest values of RSI over a given lookback period. For symmetry, we will use the same number of days in the lookback period as in the RSI calculations. You can experiment with different calculation periods if you wish. For example, if we are calculating a 14-period RSI, then we will find RSI_H and RSI_L over 14 periods. The values of stochRSI vary between +1 and zero. When RSI is at its highest value, stochRSI has a value of +1. Conversely, when RSI is at its lowest value, stochRSI is at zero. You can multiply stochRSI by 100 if you wish.

As a reminder, we can define a midpoint oscillator using RSI. The RSI%M is similar to the stochRSI but uses the midpoint of the range as its reference. Here is a midpoint oscillator definition:

$$\text{RSI\%M} = 100\frac{(2(RSI_{Today}) - RSI_{High} - RSI_{Low})}{(RSI_{High} - RSI_{Low})} \quad 21)$$

The overbought/oversold values are at +100 and -100 with RSI%M.

For MetaStock version 3.0 users, it is simple to use either Equation 20 or 21 as a custom formula. Here is the MetaStock formula for Equation 20:

22a) stochRSI = (rsi(14) - 11v(rsi(14),14)) / (hhv(rsi(14),14) - 11v(rsi(14),14))

We need to define an additional formula in MetaStock before calculating RSI%M.

22b) stochRSH = (hhv(rsi(14),14)-rsi(14)) / (hhv(rsi(14),14)-11v(rsi(14),14))

23) RSI%M = (stochRSI - stochRSH)

The values will be smoothed by MetaStock' s built-in smoothing mechanism for RSI. Now look at some data to check the relative performance of these indicators. I will use data for the Dow Jones Industrial Average (DJIA) from the first half in 1990 in Figure 12 to show a conventional 14-period RSI. The RSI said the market was oversold in late February and overbought in early March.

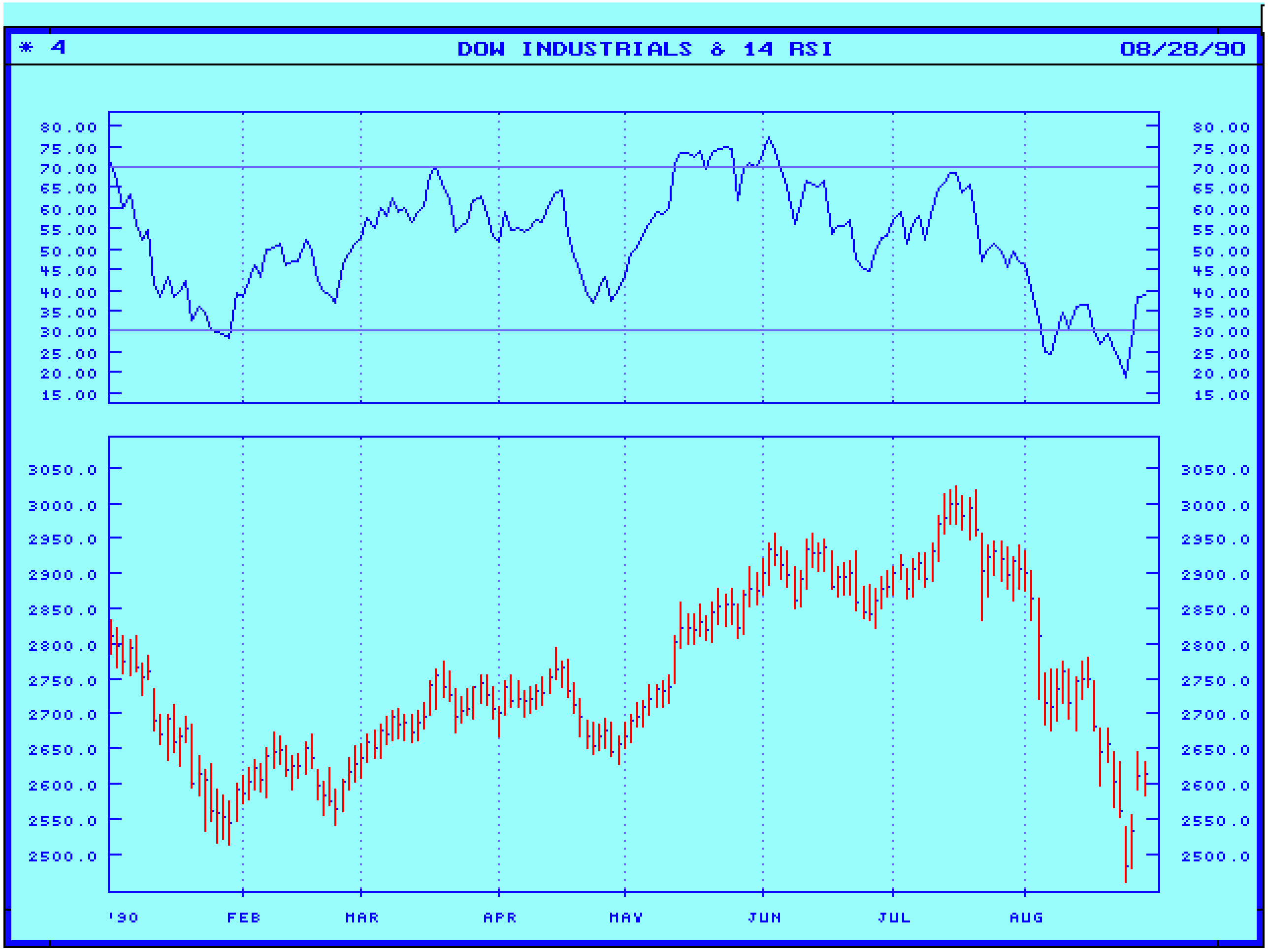

FIGURE 12: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD RSI. The market is considered oversold when the RSI is below 30 and overbought when the RSI reaches 70.

The RSI did not signal an oversold condition at the end of May. The rally into June quickly sent RSI above 70. The new peak in July did not take RSI above 70, producing aclassic divergence between RSI and the DJIA. As the DJIA slipped into August, RSI fell below 30 in early August. In late August, more selling sent the RSI to 20 before rebounding to 40.

The 14-period stochRSI is in Figure 13. This approach signals overbought/oversold signals unambiguously. For example, it signaled the oversold condition in May that the RSI did not. Similarly, it flagged the oversold condition in late June that led to the rally into July. In Figure 14 I have shown the NMO for the same period. In general, NMO flagged the extremes better than the RSI. However, the stochRSI was better than NMO. The DJIA was trending nicely in this period of comparison.

The stochRSI was the best, as it flagged each significant price extreme over this period.

Now compare relative performance with the DJIA in a trading range. We will use data from early 1991. The 14-period RSI failed to catch almost all the overbought/oversold areas (Figure 15) . The 14-period NMO was better, as it did flag two important highs and lows (Figure 16). The stochRSI (Figure 17) was the best, as it flagged each significant price extreme over this period. I have included a 14-period stochastic oscillator(Figure 18) for comparison. The stochRSI was better than the stochastic oscillator as well, as it flagged the oversold conditions in late February, late April and mid-May.

You can use the NMO for the overall view of price movement The stochRSI gives a closer look at price movement than NMO or RSI.

VARIABLE- LENGTH DMI

We have used a fixed number of days in calculating RSI in our entire discussion. For example, we used 14 days' data to calculate a 14-day RSI with a one-day lookback. This is the main reason RSI does not consistently identify overbought/oversold regions. The prices of futures contracts do not conform to an arbitrarily chosen descriptive variable. We could use RSI more effectively if we indexed the number of days to market volatility. We could then automatically adapt RSI to market action. I will call this variable-length, dynamic RSI the dynamic momentum index (DMI).

In dynamic momentum index calculations, a longer period would be used as volatility decreases. This would take a longer range view when the markets are trading quietly. Fewer periods of data would be used when volatility increases. This would shorten the horizon for finding overbought or oversold regions in active markets.

Here is one approach for indexing DMI to market volatility. First, we calculate the five-day standard deviation of closing prices. Then we take a 10-day moving average of the standard deviations. We pick the number of days in the equivalent static RSI. Let us choose 14 days for static RSI. Next, we define the following equations to calculate the number of days in the variable-length DMI.

(24) $Std_A = Average_{10}(Std(C,5))$

(25) $V_i = Std(C,5)/Std_A$

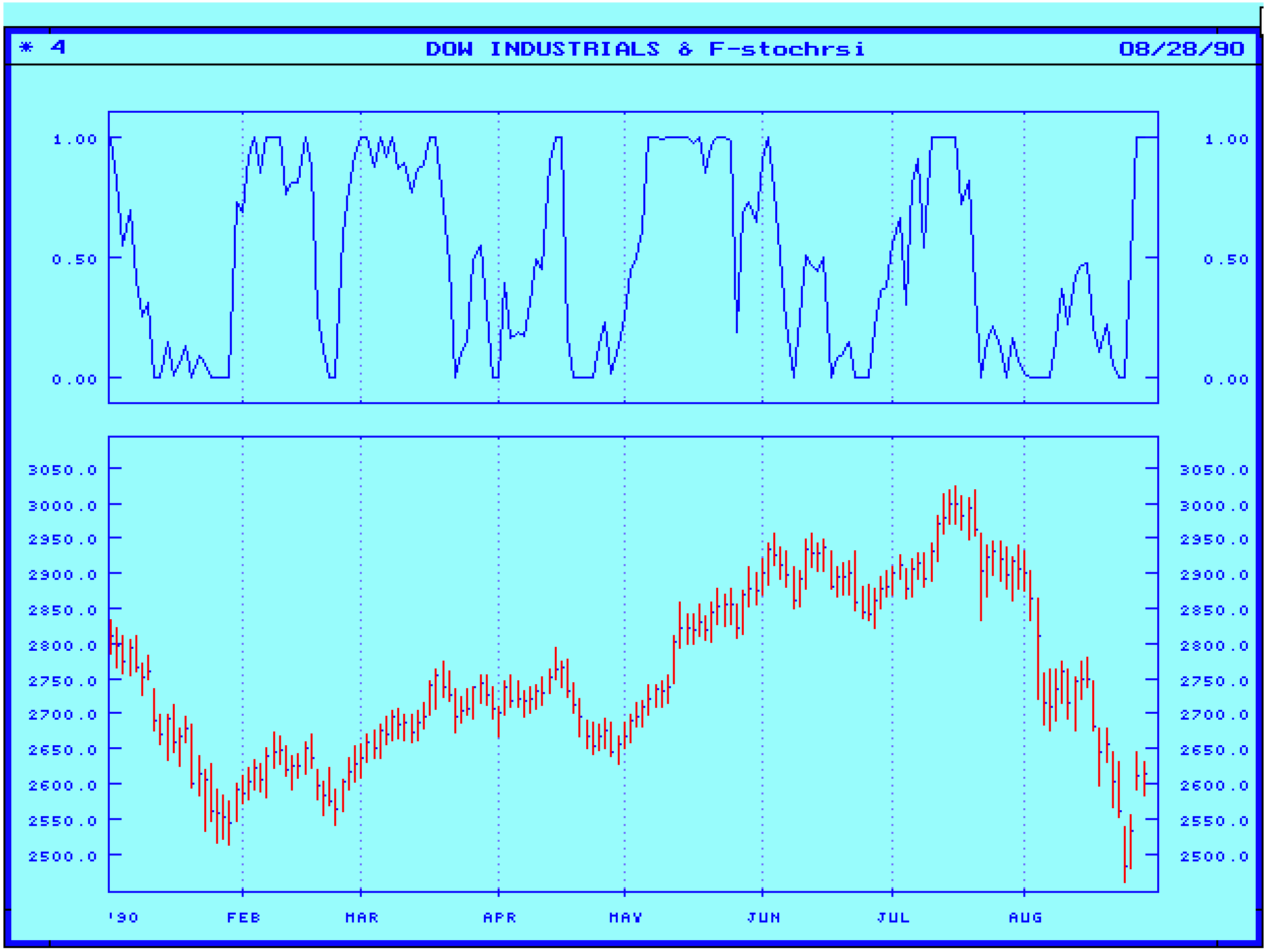

FIGURE 13: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD stochRSI. Overbought signals are +1 and oversold signals are zero for the stochRSI.

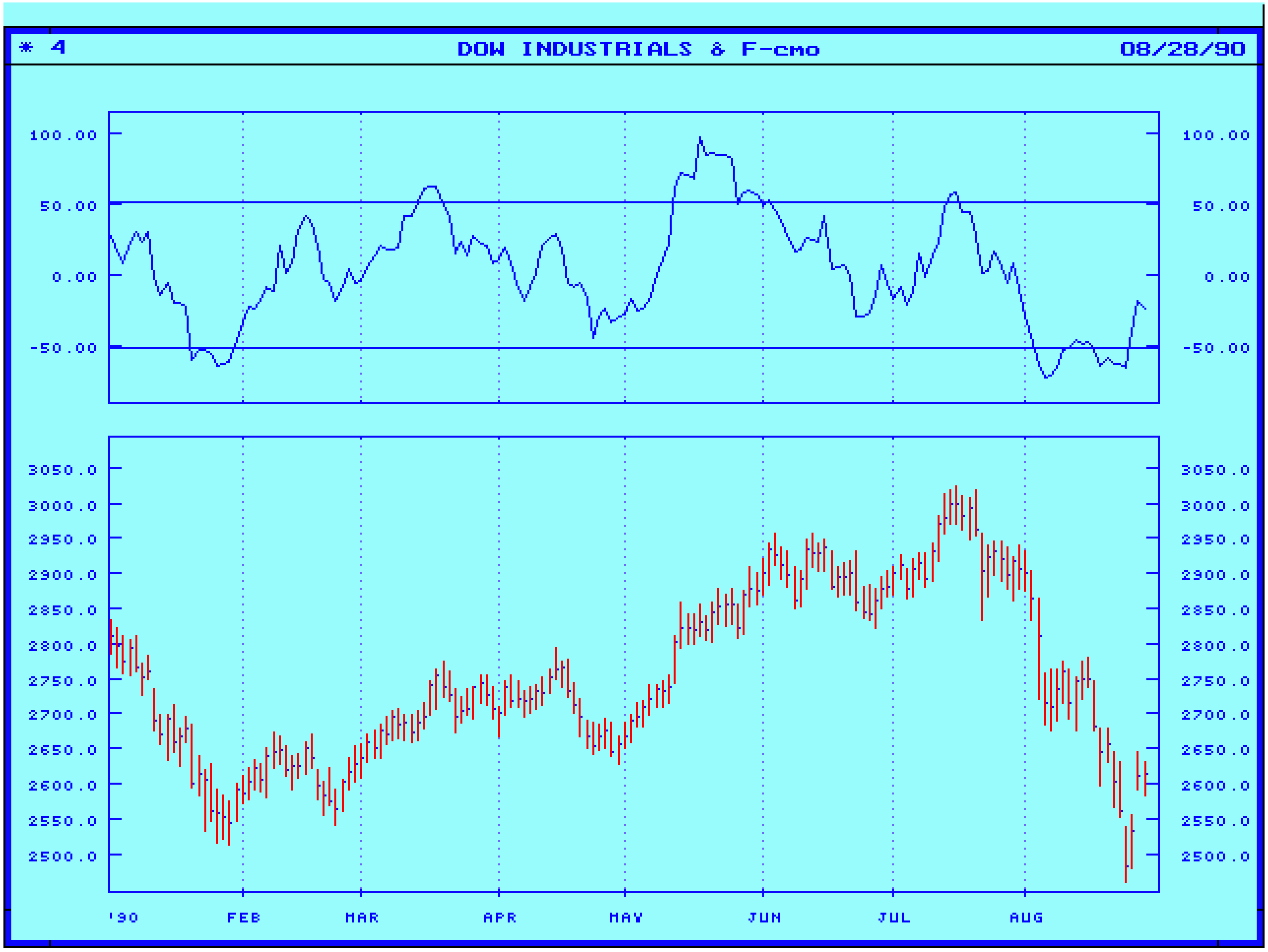

FIGURE 14: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD NET MOMENTUM OSCILLATOR. Overbought signals are +50 and oversold signals are -50 for the NMO.

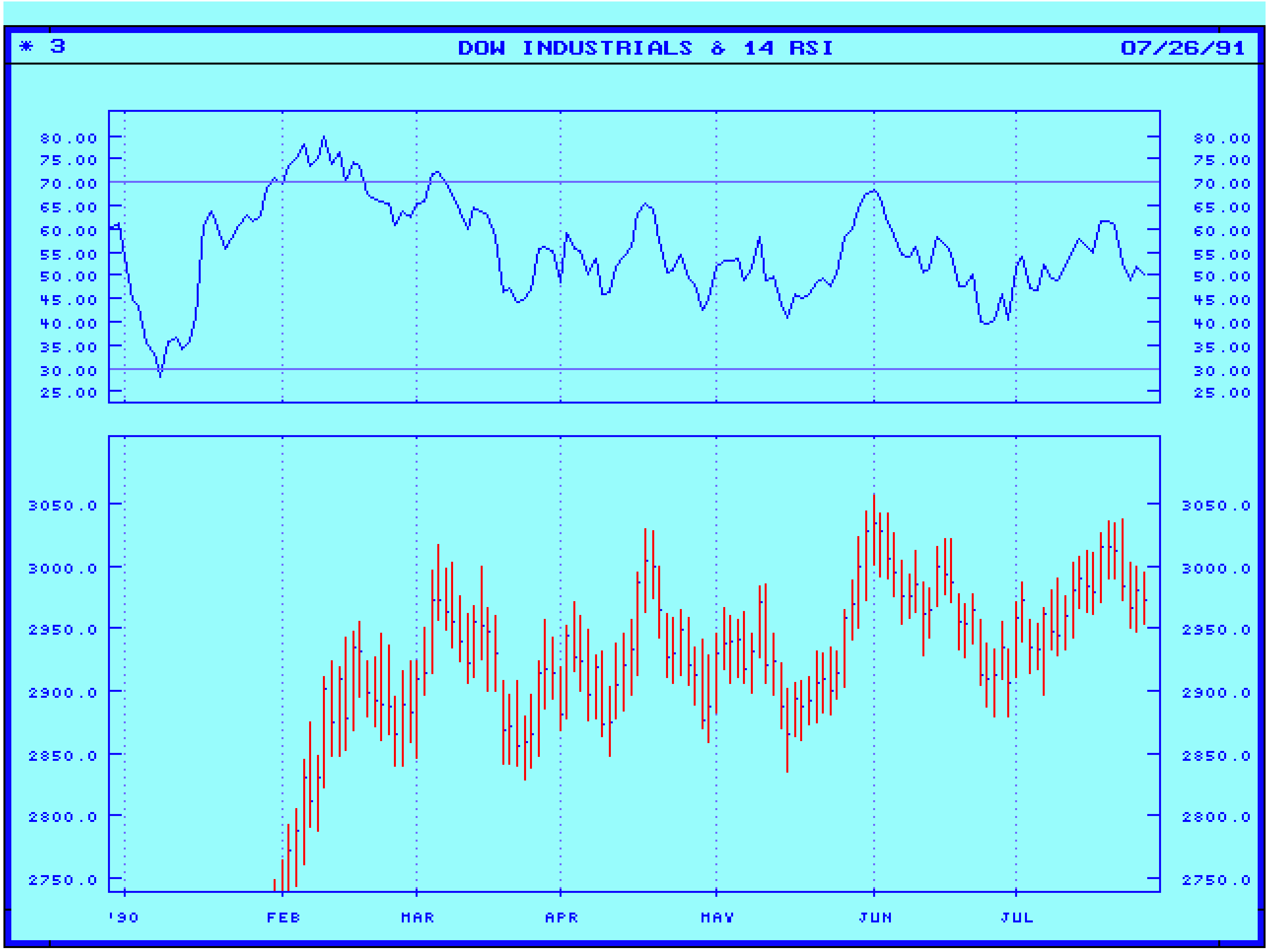

FIGURE 15: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD RSI. During the trading range from late March to July, the RSI never reached overbought or oversold values.

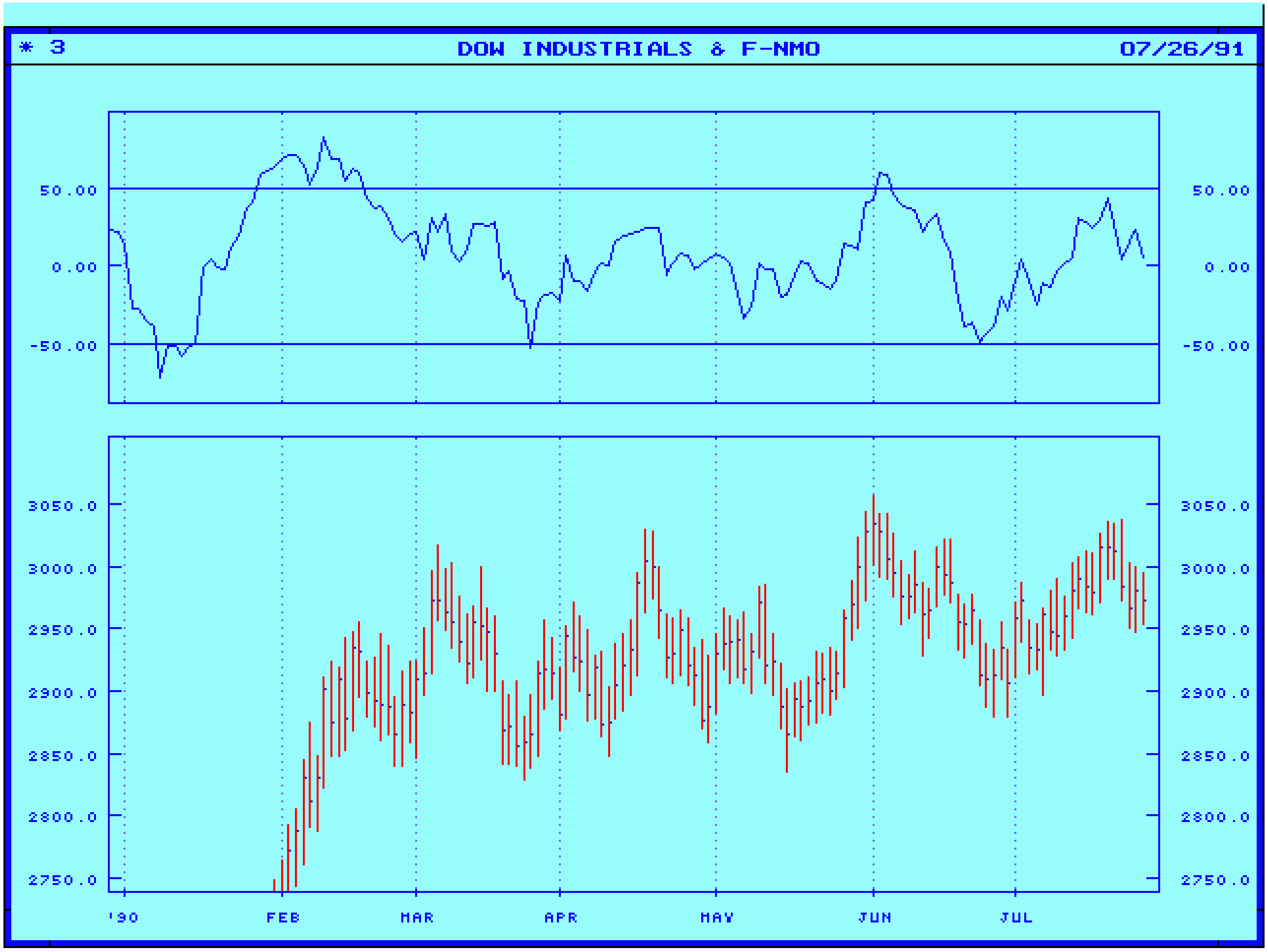

FIGURE 16: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD NMO. The NMO reached oversold levels in late March and overbought levels in early June.

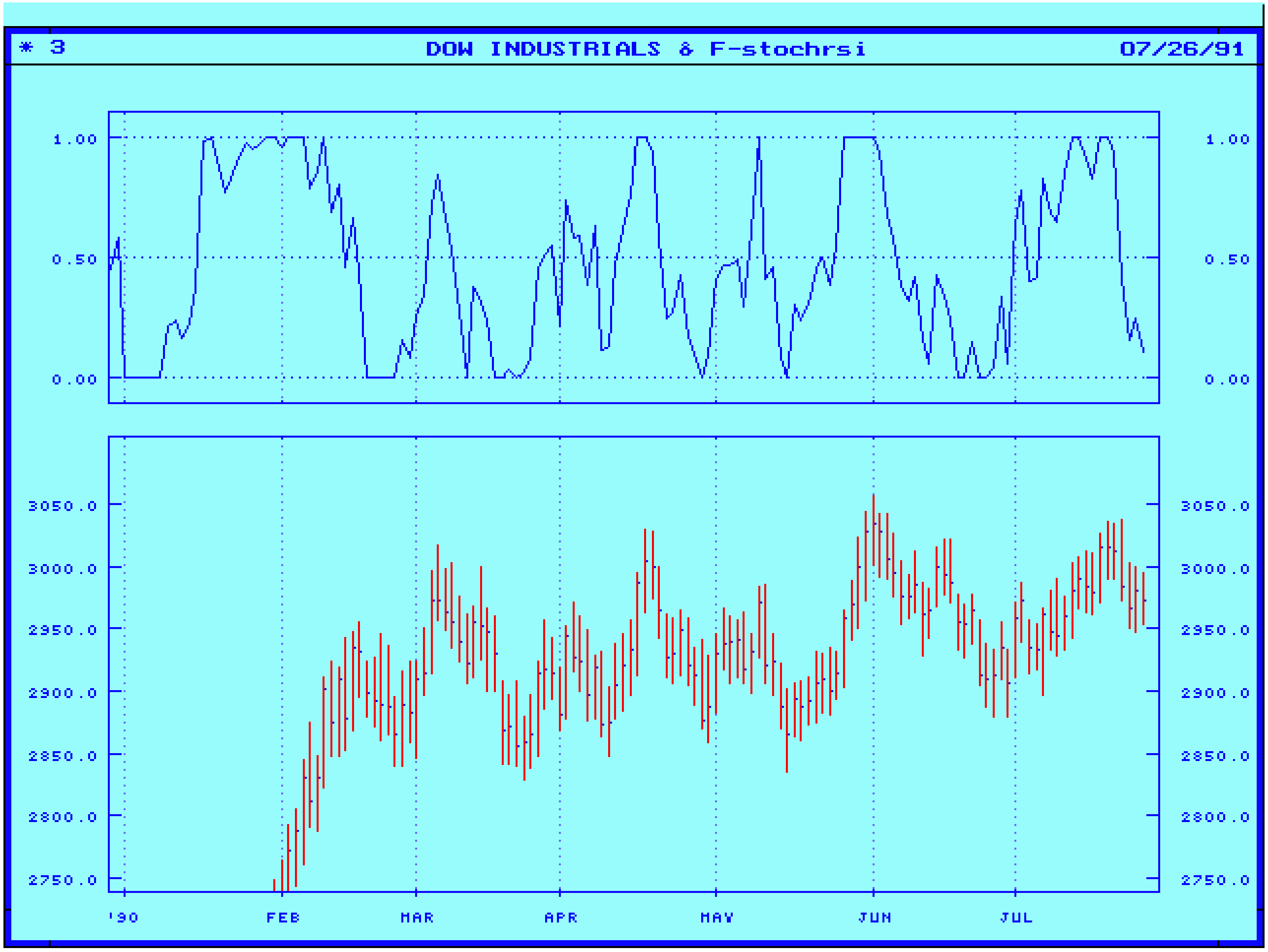

FIGURE 17: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD stochRSI. The stochRSI flagged each price extreme.

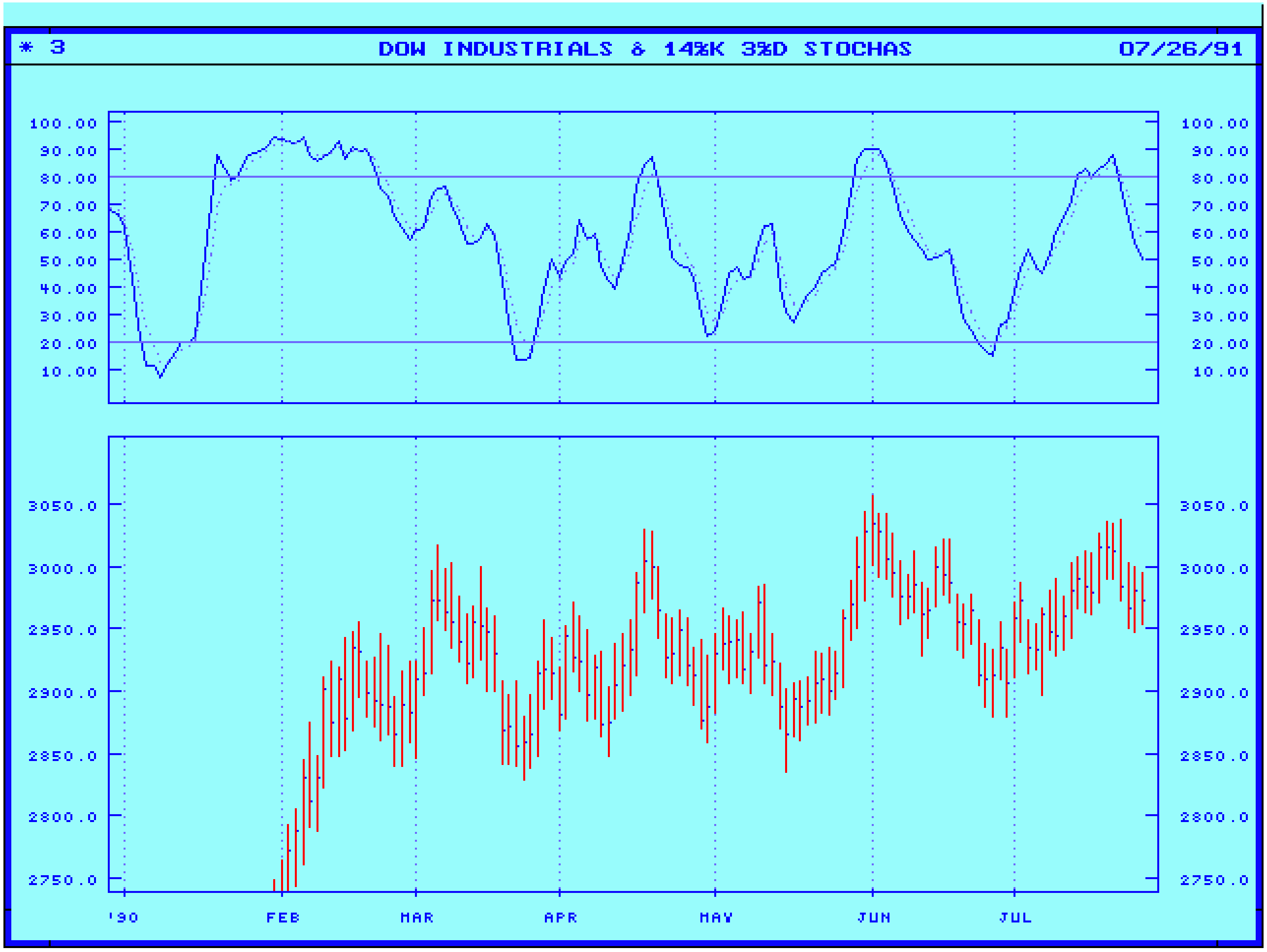

FIGURE 18: DOW JONES INDUSTRIAL AVERAGE AND THE 14-PERIOD STOCHASTICS. Here, the stochastics using a 14%K and a 3%D is presented for comparison.

(26) $T_D = INT(14/V_i)$

In Equation 24, the Std_A is a 10-day simple moving average of the five-day standard deviation of the close. The volatility index (V_i) is the ratio of today's value of the five-day standard deviation divided by its average value over the past 10 days. As volatility increases, V_i increases. The dynamic time period for RSI calculations T_D divides 14 by the volatility index. The notation INT ensures that we use integer values for the number of days in DMI calculations. If the index is greater than 1, then T_D is less than 14. Thus, a volatility increases, the number of days in DMI calculations decrease. If the index is less than 1, then T_D increases. We also define lower and upper bounds for the number of days in the calculations:

(27) $(T_D)_{MAX} = 30$

(28) $(T_D)_{MIN} = 5$

We have arbitrarily restricted the maximum and minimum number of days used for DMI calculations to 30 and 5. The reasoning here is that these limits fit our trading horizon. You can use other limits as you wish. The volatility in the data will determine choice of the limits.

In Figure 19 you can see how the volatility index converts into the variable length of the DS. When the index drops to 0.47, the number of days increase to 30. When the index increases to 2.80, the number of days decrease to five.

We should note that the conversion is nonlinear. The nonlinearity arises from the definition of the volatility index itself. The values of the percentage change in V_i will give you a sense of the nonlinearity. The conversion is more sensitive when the index is less than 1. The conversion is less sensitive to changes when the index is greater than 2.

We have built the variable-length DMI around the static RSI length of 14 days. Hence, when the V_i is approximately 1, the DMI and RSI will have similar values. As the index drops below 1, DMI and RSI values will diverge quickly. As the index increases above 1, DMI and RSI values will diverge slowly. Hence, the exact nature of the DMI-RSI curves will depend on the volatility in the data over the given test period. The divergence will increase as volatility increases. Implicitly, we are discussing unsmoothed RSI values. The smoothing mechanism used for RSI will influence the actual divergence between DMI and RSI. We recommend using all available data to calculate DMI. Then smooth DMI with a simple or exponential moving average.

We will use DJIA data from the trading range in early 1991 to show how the length of DMI varies. Figure 20 shows the scaled length of DMI plotted below the DJIA daily close. We multiplied the days in the DMI by 10 and added the result to 2550 to get the scaled values, allowing you to directly compare the length of DMI with market action. Every period of market volatility reduces the length of DMI. When the market traded quietly, the length of DMI increased. Note the short length used for DMI calculations in early March, late March, mid-April, late May and mid-June. In each instance, the market was making quick moves, up or down.

The actual values of unsmoothed Dmi and RSI are in Figure 21. The corresponding daily DJIA close is in Figure 20. The shorter length helps identify market extremes. Note the connection between the short length of DMI and the corresponding values of DMI. Overbought and oversold values occurred when we were using a length of DMI less than 14 days. The variable-length dynamic momentum index flagged

CONVERTING VOLATILITY INDEX INTO VARIABLE LENGH FOR CALCULATING DMI

V_i	Change in V_i RATIO(%)	T_D=INT(14/V_i) (DAYS)
0.47		30.00
0.50	6.38	28.00
0.70	40.00	20.00
0.90	28.57	16.00
1.10	22.22	13.00
1.30	18.18	11.00
1.50	15.38	9.00
1.70	13.33	8.00
2.10	23.52	7.00
2.50	19.04	6.00
2.80	10.71	5.00

FIGURE 19: *An increase in the volatility ratio shortens the number of days data for calculating the RSI and a decrease in the volatility ratio increases the number of days data for calculating the RSI.*

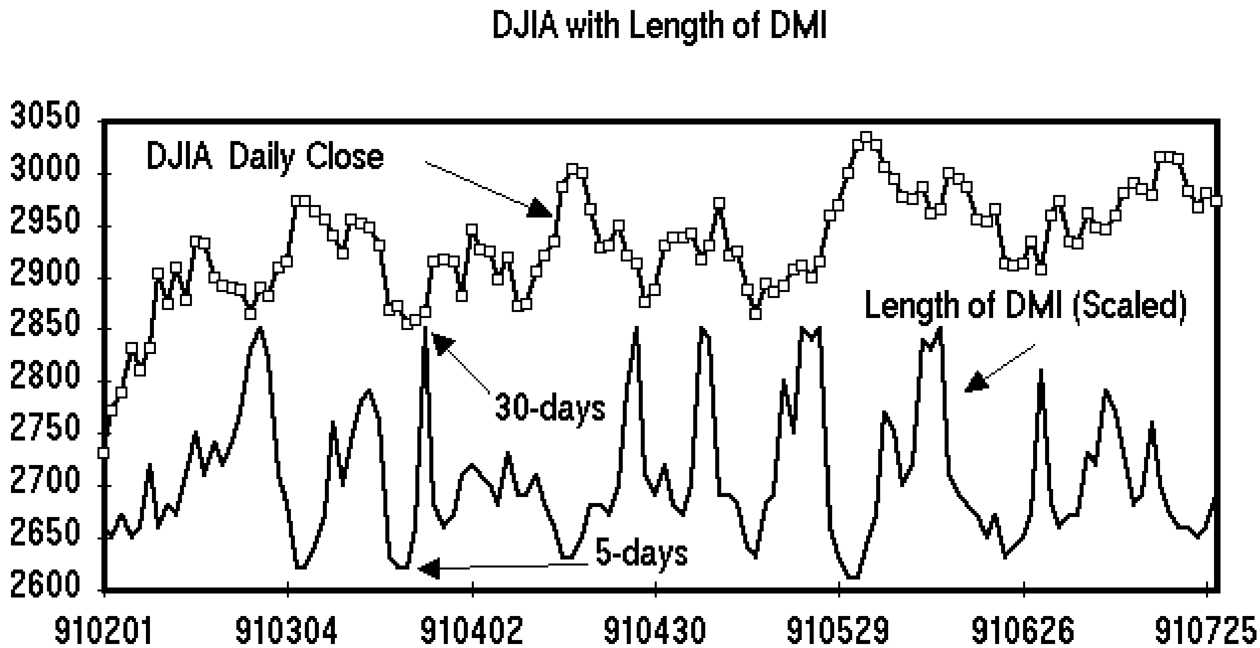

FIGURE 20: DJIA AND DYNAMIC MOMENTUM INDEX. The variable-length dynamic momentum index is the RSI with the lookback period adjusted based on market volatility. The more volatile the market is, the shorter the lookback period.

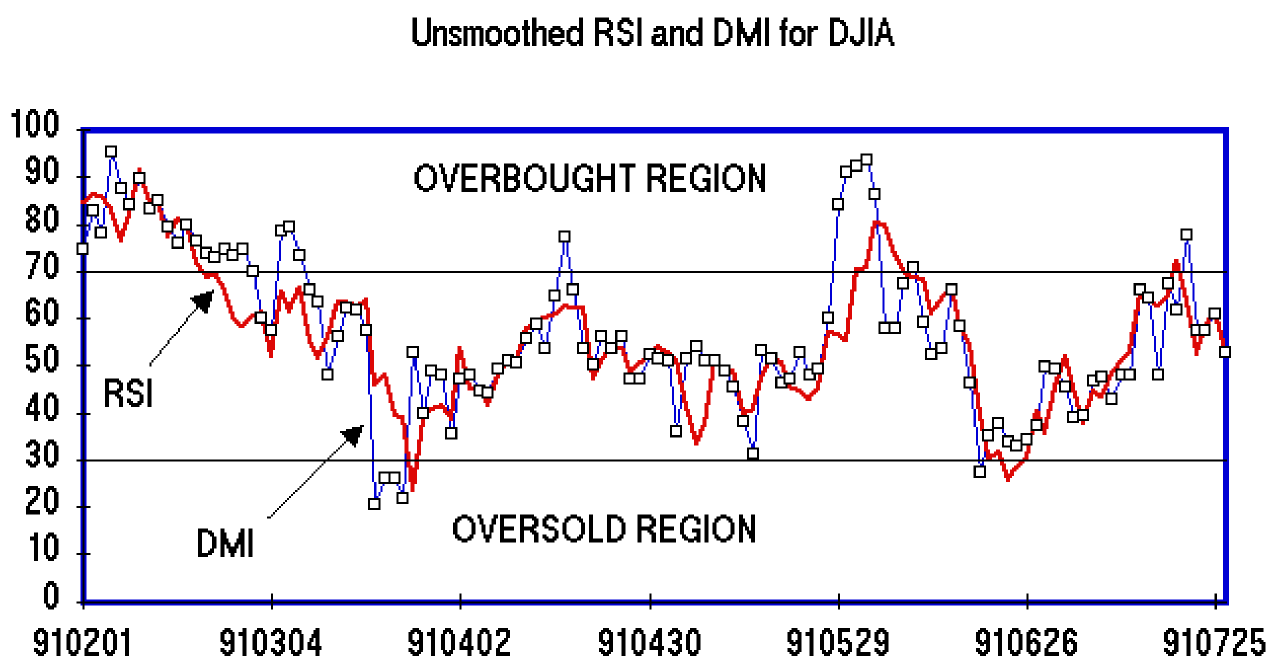

FIGURE 21: UNSMOOTHED RSI AND THE DMI FOR THE DJIA. The DMI is more sensitive to price extremes than the RSI.

overbought conditions based on market action. This differs from the stochastic RSI approach, which flags extremes in values of RSI. As expected, the variable length DMI is more sensitive to price changes than the static RSI. You can verify this by noting when the DMI and RSI reached the overbought or oversold regions in Figure 21. The DMI leads RSI by several days in noting extremes. The DMI led the RSI by four days in Figure 21. Sometimes (see mid-April), the DMI reached price extremes, while the RSI did not.

BY ANY OTHER NAME

In summary, this tutorial should give you agood feel for the details of the calculations of RSI, RMI, NMO and TSI. All show momentum in one form or another. We can use relative and absolute momentum to understand what they are saying. I recommend using all data to calculate new values of these indicators. You can then smooth the "unsmoothed" values with a moving average for better representation of market action.

The net momentum oscillator elegantly shows the ratio of relative momentum to absolute momentum over the chosen period. It fluctuates between -100 and + 100 and uses the ratio of up day momentum and down day momentum to define overbought and oversold regions. A ratio of 3:1 or 1:3 of the two components gives overbought and oversold levels of +50 and -50. Thus, it may be easier to interpret. It seems to flag extremes better than the conventional RSI. We calculated a table for converting RSI and NMO values into values for the ratio of S_u/S_d. You can use this ratio to track buying pressure or selling pressure.

The NMO and TSI are similar. TSI uses double-smoothed exponential averages in its calculations and shows greater sensitivity to price changes but may not show extremes in momentum as well as NMO. We cannot unambiguously estimate the ratio of S_U,/S_d for TSI because of exponential smoothing.

The stochastic RSI approach was best at signaling extremes in prices. You can also define this oscillator by using the midpoint of the range of values. This is a powerful combination of two popular approaches that reduces redundancy. This function can be easily displayed in most technical analysis programs.

The variable-length dynamic momentum index (DMI) is another useful variant of RSI. We have indexed DMI to market volatility and automatically adjusts to market action. This is also a powerful extension of RSI calculations.

The purpose of all these calculations is to measure momentum. In all, using them can make momentum work for you.

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THE RSI AND THE RMI

The formula for the relative strength index (RSI) is:

RSI = 100((RS/(1+RS))

where:
RS = AU/AD during the last X days
AU = Yesterday's AU - (AU/X) + today's up close or zero
AD = Yesterday's AD - (AD/X) + the absolute value of today's down close or zero

The formula for the relative momentum index (RMI) is:

RMI = 100((RM/(1+RM))

where:
RM = MU/MD during the last X days
MU = Yesterday's MU - (MU/X) + today's difference between today's close and the close Y days ago if positive, or zero
MD = Yesterday's MD - (MD/X) + the absolute value of today's difference between today's close and the close Y days ago if negative, or zero

Calculating the RMI in Excel 4.0, using the parameters y = 5 and x = 20 (sidebar Figure 1): Column C measures the difference between today's price and the price five days ago. If the price today is higher than the price five days ago, the difference is returned in the cell. If the difference is negative, then a zero is returned. The formula for cell C7 is:

=IF(B7>B2,B7-B2,0)

Column D returns the absolute value of the difference between today's price and the price five days ago if the difference is negative, otherwise a zero is returned. The formula for cell D7 is:

=IF(B7<B2,ABS(B7-B2),0)

Column E is the smoothed sum of the last 20 days' values in column C. First, we calculate the sum of the last 20 days' values from column C. The formula for cell E26 is:

=SUM(C7:C26)

The remaining cells in column C use an averaging technique to smooth the data. Simply subtract 1/20 of yesterday's value and add today's value from column C. The formula for cell E27 is:

=E26-(E26/20)+C27

Column F is the smoothed sum of the values in column D. Again, first calculate the sum of the last 20 days' values for column D. The formula for cell F26 is:

=SUM(D7:D26)

The remaining cells use the average off technique. The formula for cell F27 is:

=F26-(F26/20)+D27

Column G is the ratio of column E and column F. The formula for cell G27 is:

=E27/F27

The relative momentum index is calculated in column H. The formula for cell H27 is:

=100*(G27/(1+G27))

—Editor

	A	B	C	D	E	F	G	H
1	Date	S&P 500						
2	11/29/91	375.22						
3	12/2/91	381.40						
4	12/3/91	380.96						
5	12/4/91	380.07						
6	12/5/91	377.39						
7	12/6/91	379.10	3.88	0				
8	12/9/91	378.26	0	3.14				
9	12/10/91	377.90	0	3.06				
10	12/11/91	377.70	0	2.37				
11	12/12/91	381.55	4.16	0				
12	12/13/91	384.47	5.37	0				
13	12/16/91	384.46	6.2	0				
14	12/17/91	382.74	4.84	0				
15	12/18/91	383.48	5.78	0				
16	12/19/91	382.52	0.97	0				
17	12/20/91	387.04	2.57	0				
18	12/23/91	396.82	12.36	0				
19	12/24/91	399.33	16.59	0				
20	12/26/91	404.84	21.36	0				
21	12/27/91	406.46	23.94	0				
22	12/30/91	415.14	28.1	0				
23	12/31/91	417.09	20.27	0				
24	1/2/92	417.26	17.93	0				
25	1/3/92	419.34	14.5	0				
26	1/6/92	417.96	11.5	0	200.32	8.57	23.37	95.90
27	1/7/92	417.40	2.26	0	192.56	8.14	23.65	95.94
28	1/8/92	418.10	1.01	0	183.95	7.73	23.78	95.96
29	1/9/92	417.61	0.35	0	175.10	7.35	23.83	95.97
30	1/10/92	415.10	0	4.24	166.34	11.22	14.83	93.68

SIDEBAR FIGURE 1